A causal interpretation of EPR-B experiment

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Abstract

In this paper we study a two-step version of EPR-B experiment. Its theoretical resolution in space and time enables us to refute the classic "impossibility" to decompose a pair of entangled atoms into two distinct states, one for each atom. We propose a new causal interpretation of the EPR-B experiment where each atom has a position and a spin while the singlet wave function verifies the two-body Pauli equation.

1 Introduction

The nonseparability is one of the most puzzling aspects of quantum mechanics. For over thirty years the spin version proposed by Bohm [5, 6] of the Einstein-Podolsky-Rosen experiment [1], the Bell theorem [2] and the BCHSH inequalities [2, 3, 4] have been at the heart of the debate on hidden variables and non-locality; but hitherto the precise nature of the physical process that lies behind the "non-local" correlations in the spins of the particles has remained unclear.

Many experiments since Bell's paper have demonstrated violations of these inequalities and have vindicated quantum theory [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The first one was done with pairs of entangled photons and clearly violate Bell's inequality [10, 11, 12, 13]. Entangled protons have also been studied in an early experiment [9]. The generation of EPR pairs of massive atoms instead of massless photons has been considered [14, 15]; it also shows experimental violation of Bell's inequality with efficient detection [15].

In a new experiment, Zeilinger and all [26] measure previously untested correlations between two entangled photons, they show that these correlations violate an inequality proposed by Leggett for non-local realistic theories [25].

The usual conclusion of these experiments is to reject the non-local realism because the impossibility to decompose a pair of entangled atoms into two states, one for each atom.

In this paper we show that this decomposition is possible: a causal interpretation exists where each atom has a position and a spin while the singlet wave function verifies the two-body Pauli equation.

To demonstrate this; we consider a two-step version of EPR-B experiment and we use an analytic expression of the wave function and the probability density. The explicit solution is obtained via a complete integration of the two-body Pauli equation *over time and space.*

A first causal interpretation of EPR-B experiment was proposed in 1987 by Dewdney, Holland and Kyprianidis [21, 22]. This interpretation had a flaw: the spin module of each particle varied during the experiment from 0 to $\frac{\hbar}{2}$. It is perhaps the cause of its rejection by the scientific community as an ad hoc solution.

The explicit solution in terms of two-body Pauli spinors and the probability density for the two steps of the EPR-B experiment are pesented in section 2. The solution in space and time shows how it is possible to deduce tests on the spatial quantization of particles, similar to those of the Stern and Gerlach experiment.

In section 3, we provide a realistic explanation of the entangled states and a method to desentangle the wave function of the two particles.

The resolution in space of the equation Pauli is essential: it enables the spatial quantization in section 2 and explains determinism and desentangling in section 3.

2 Simulation and tests of EPR-B experiment in two steps

Fig.1 presents the Einstein-Podolsky-Rosen-Bohm experiment. A source S created in O pairs of identical atoms A and B, but with opposite spins. The atoms A and B split following 0y axis in opposite directions, and head towards two identical Stern-Gerlach apparatus $\mathcal{A}$ and $\mathcal{B}$.

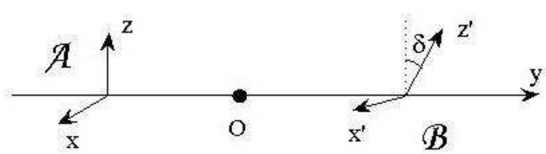

Figure 1: Schematic configuration of EPR-B experiment.

The electromagnet $\mathcal{A}$ "measures" the A spin in the direction of the Oz-axis and the electromagnet $\mathcal{B}$ "measures" the B spin in the direction of the Oz'-axis, which is obtained after a rotation of an angle δ around the Oy-axis.

We further consider that atoms A and B may be represented by Gaussian wave packets in x and z. The initial wave function of the entangled state is the singlet state:

$$\Psi_0(x_A, z_A, x_B, z_B) = f(x_A, z_A) f(x_B, z_B) \frac{1}{\sqrt{2}} (|+_A\rangle|-_B\rangle - |-_A\rangle|+_B\rangle) \qquad (1)$$

where for example $f(x,z) = (2\pi\sigma_0^2)^{-\frac{1}{2}} e^{-\frac{x^2+z^2}{4\sigma_0^2}}$ and where $|\pm_A\rangle$ and $|\pm_B\rangle$ are the eigenvectors of the operators σ_{z_A} and σ_{z_B}: $\sigma_{z_A}|\pm_A\rangle = \pm|\pm_A\rangle$, $\sigma_{z_B}|\pm_B\rangle = \pm|\pm_B\rangle$. We treat classically dependence with y: speed $-v_y$ for A and v_y for B.

The wave function of the two identical particles A and B, electrically neutral and with magnetic moments μ_0, subject to magnetic fields, admits 4 components $\Psi^{i_1,i_2}(\mathbf{r}_1, \mathbf{r}_2, t)$ and verifies the two-body Pauli equation:

$$i\hbar \frac{\partial \Psi^{i_1,i_2}}{\partial t} = (-\frac{\hbar^2}{2m}\Delta_A - \frac{\hbar^B}{2m}\Delta_2)\Psi^{i_1,i_2} + \mu B_{Ai}(\sigma_i)^{i_1}_j \Psi^{j,i_2} + \mu B_{Bi}(\sigma_i)^{i_2}_k \Psi^{i_1,k} \quad (2)$$

with the initial conditions:

$$\Psi^{i_1,i_2}(\mathbf{r}_1, \mathbf{r}_2, 0) = \Psi_0^{i_1,i_2}(\mathbf{r}_1, \mathbf{r}_2) \quad (3)$$

where the σ_i are the Pauli matrixes and where the $\Psi_0^{i_1,i_2}(\mathbf{r}_1, \mathbf{r}_2)$ are given functions.

In the basis $|\pm_A\rangle$ et $|\pm_B\rangle$ the initial condition $\Psi_0^{i_1,i_2}(\mathbf{r}_1, \mathbf{r}_2)$ corresponds to the singlet state (1).

We take as numerical values those of the Stern-Gerlach experiment with silver atoms [18, 19]. For a silver atom, one has $m = 1,8 \times 10^{-25}$ kg, $v_y = 500$ m/s , σ_0=10^{-4}m. For the electromagnetic field $\mathbf{B}$, $B_x = B'_0 x$; $B_y = 0$ and $B_z = B_0 - B'_0 z$ with $B_0 = 5$ Tesla, $B'_0 = \left|\frac{\partial B}{\partial z}\right| = -\left|\frac{\partial B}{\partial x}\right| = 10^3$ Tesla/m over a length $\Delta l = 1$ *cm*. The screen that intercepts atoms is at a distance $D = 20$ *cm* (time $t = \frac{D}{v_y} = 4 \times 10^{-4}$s) from the exit of the magnetic field.

One of the difficulties of the interpretation of the EPR-B experiment is the existence of two simultaneous measurements. By doing these measurements one after the other, the interpretation of the experiment will be facilitated. That is the purpose of the two-step version of the experiment EPR-B studied below.

2.1 First step: Measurement of A spin and position of B

In the first step we make, on a couple of particles A and B in a singlet state, a Stern and Gerlach "measurement" for atom A, and for atom B a mere impact measurement on a screen.

It is the experiment first proposed in 1987 by Dewdney, Holland and Kyprianidis [21].

Consider that at time t_0 the particle A arrives at the entrance of electromagnet (A). The wave function at time $t_0 + t$ can be calculated, from the wave function (1), term to term in basis $[|\pm_A\rangle, |\pm_B\rangle]$. After the exit of the magnetic field $\mathcal{A}$, at time $t_0 + t + \triangle t$, the wave function (1) becomes [19]:

$$\Psi(x_A, z_A, x_B, z_B, t_0 + t + \triangle t) = \frac{1}{\sqrt{2}} f(x_B, z_B)[\quad f^+(x_A, z_A, t)|+_A\rangle|-_B\rangle \quad (4)$$
$$- \quad f^-(x_A, z_A, t)|-_A\rangle|+_B\rangle]$$

with

$$f^\pm(x, z, t) = f(x, z \mp z_\triangle \mp ut) e^{i(\frac{\pm muz}{\hbar} + \varphi^\pm(t))} \quad (5)$$

and

$$\Delta t = \frac{\Delta l}{v_y} = 2 \times 10^{-5} s, \qquad z_\Delta = \frac{\mu_0 B'_0 (\Delta t)^2}{2m} = 10^{-5} m,$$

$$u = \frac{\mu_0 B'_0 (\Delta t)}{m} = 1 m/s. \tag{6}$$

The atomic density $\rho(z_A, z_B, t_0 + t + \Delta t)$ is found by integrating $\Psi(x_A, z_A, x_B, z_B, t_0 + t + \triangle t)^* \Psi(x_A, z_A, x_B, z_B, t_0 + t + \triangle t)$ on x_A and x_B:

$$\begin{aligned} \rho(z_A, z_B, t_0 + t + \Delta t) &= (2\pi\sigma_0^2)^{-\frac{1}{2}} e^{-\frac{(z_B)^2}{2\sigma_0^2}} \qquad (7) \\ &\quad \times (2\pi\sigma_0^2)^{-\frac{1}{2}} \frac{1}{2} \left(e^{-\frac{(z_A - z_\Delta - ut)^2}{2\sigma_0^2}} + e^{-\frac{(z_A + z_\Delta + ut)^2}{2\sigma_0^2}} \right). \end{aligned}$$

We deduce that the beam of particles A is divided into two, while the B beam of particle stays one. This result can easily be tested experimentally.

Moreover, we note that the space quantization of particle A is identical to that of an untangled particle in a Stern and Gerlach apparatus: the distance $\delta z = 2(z_\Delta + ut)$ between the two spots N^+ (Spin $+$) and N^- (spin $-$) of a family of particle A is the same as the distance between the two spots N^+ and N^- of a particle in a classic Stern and Gerlach experiment [19]. This result can easily be tested experimentally.

We finally deduce from (7) that :

- the density of A is the same, whether particle A is entangled with B or not,
- the density of B is not affected by the "measurement" of A.

These two predictions of quantum mechanics can be tested. Only spins are involved. We conclude from (4) that the spins of A and B remain opposite throughout the experiment.

2.2 Second step : "Measurement" of A spin, then of B spin.

The second step is a continuation of the first and results in realizing the EPR-B experiment in two steps.

On a couple of particles A and B in a singlet state, first we made a Stern and Gerlach "measurement" on the A atom between t_0 and $t_0 + t + \triangle t$, then a Stern and Gerlach "measurement" on the B atom with an electromagnet $\mathcal{B}$ forming an angle δ with $\mathcal{A}$ between $t_0 + t + \triangle t$ and $t_0 + 2(t + \triangle t)$.

Beyond the exit of magnetic field $\mathcal{A}$, at time $t_0 + t + \triangle t$, the wave function is given by (4). Immediately after the "measurement" of A, still at time $t_0+t+\triangle t$, if the A measurement is $\pm$, the conditionnal wave functions of B are:

$$\Psi_{B/\pm A}(x_B, z_B, t_0 + t + \triangle t) = f(x_B, z_B) | \mp B\rangle. \tag{8}$$

To measure B, we refer to the basis $|\pm' B\rangle$. So, after the measurement of B, at time $t_0 + 2(t + \triangle t)$ the conditional wave functions of B are :

$$\begin{aligned}
\Psi_{B/+A}(x'_B, z'_B, t_0 + 2(t+\triangle t)) &= \cos\frac{\delta}{2} f^+(x'_B, z'_B, t)|+'_B\rangle \\
&\quad + \sin\frac{\delta}{2} f^-(x'_B, z'_B, t)|-'_B\rangle, \\
\Psi_{B/-A}(x'_B, z'_B, t_0 + 2(t+\triangle t)) &= -\sin\frac{\delta}{2} f^+(x'_B, z'_B, t)|+'_B\rangle \\
&\quad + \cos\frac{\delta}{2} f^-(x'_B, z'_B, t)|-'_B\rangle.
\end{aligned}$$

We therefore obtain, in this two steps version of the EPR-B experiment, the same results for spatial quantization and correlations of spins as in the EPR-B experiment.

3 Causal interpretation of the EPR-B experiment

We assume, at moment of the creation of the two entangled particles A and B, that each of the two particles A and B has an initial wave function $\Psi^0_A(\mathbf{r}_A, \theta^0_A, \varphi^0_A)$ and $\Psi^0_B(\mathbf{r}_B, \theta^0_B, \varphi^0_B)$ with spinors which are opposite spins; for example : $\Psi^0_A(\mathbf{r}_A, \theta^0_A, \varphi^0_A) = f(\mathbf{r}_A)(\cos\frac{\theta^0_A}{2}|+_A\rangle + \sin\frac{\theta^0_A}{2} e^{i\varphi^0_A}|-_A\rangle)$ and $\Psi^0_B(\mathbf{r}_B, \theta^0_B, \varphi^0_B) = f(\mathbf{r}_B)(\cos\frac{\theta^0_B}{2}|+_B\rangle + \sin\frac{\theta^0_B}{2} e^{i\varphi^0_B}|-_B\rangle)$ with $\theta^0_B = \pi - \theta^0_A$ and $\varphi^0_B = \varphi^0_A - \pi$.

Then the Pauli principle tells us that the two-body wave function must be antisymmetric; after calculation we find:

$$\Psi^0(\mathbf{r}_A, \theta_A, \varphi_A, \mathbf{r}_B, \theta_B, \varphi_B) = -e^{i\varphi_A} f(\mathbf{r}_A) f(\mathbf{r}_B)(|+_A\rangle|-_B\rangle - |-_A\rangle|+_B\rangle)$$

which is the same as the singlet state, factor wise (1).

Thus, we can consider that the singlet wave function is the wave function of a family of two fermions A and B with opposite spins: direction of initial spin A and B exist, but is not *known*. It is a local hidden variable which is therefore necessary to add in the initial conditions of the model.

This is not the interpretation followed by the school of Bohm [21, 22, 24, 23] in the interpretation of the singlet wave function; they suppose, for example, a zero spin for each of particles A and B at the initial time.

It remains to determine the wave function and the trajectories of particles A and B: from the entangled wave function, initial spins and initial positions of each particle.

We assume therefore that the intial position of the particle A is known $(x^A_0, y^A_0 = 0, z^A_0)$ as well as the particle B $(x^B_0 = x^A_0, y^B_0 = y^A_0 = 0, z^B_0 = z^A_0)$.

3.1 Step 1 : Measurement of A spin and position of B

Equation (4) shows that the spins of A and B remain opposite throughout step 1. Equation (7) shows that the densities of A and B are independent; for A equal to the density of a family of free particles in a classical Stern Gerlach apparatus, whose initial spin orientation has been randomly chosen; for B equal to the density of a family of free particles.

The spin of a particle A is orientated gradually following the position of the particle in its wave into a spin + or −. The spin of particle B follows that of A, while remaining opposite.

In the equation (4) particle A can be considerd independent of B. We can therefore give it the wave function

$$\Psi_A(x_A, z_A, t_0 + t + \triangle t) = \cos\frac{\theta_A^0}{2} f^+(x_A, z_A, t)|+_A\rangle + \sin\frac{\theta_A^0}{2} e^{i\varphi_A^0} f^-(x_A, z_A, t)|-_A\rangle \quad (9)$$

which is that of a free particle in a Stern Gerlach apparatus and whose initial spin is given by $(\theta_A^0, \varphi_A^0)$.

So, the equation of its trajectory is given by the following differential equations: in the interval $[t_0, t_0 + \Delta t]$:

$$\frac{dz_A}{dt} = \frac{\mu_0 B_0' t}{m} cos\theta(z_A, t) \text{ with } \tan\frac{\theta(z_A, t)}{2} = \tan\frac{\theta_0}{2} e^{-\frac{\mu_0 B_0' t^2 z_A}{2m\sigma_0^2}} \quad (10)$$

with the initial condition $z_A(t_0) = z_0^A$; and in the interval $t_0 + \Delta t + t$ $(t \geq 0)$:

$$\frac{dz_A}{dt} = u\frac{\tanh(\frac{(z_\Delta + ut)z_A}{\sigma_0^2}) + \cos\theta_0}{1 + \tanh(\frac{(z_\Delta + ut)z_A}{\sigma_0^2})\cos\theta_0}$$

$$\text{et} \quad \tan\frac{\theta(z_A(t), t)}{2} = \tan\frac{\theta_0}{2} e^{-\frac{(z_\Delta + ut)z_A}{\sigma_0^2}}. \quad (11)$$

$\theta(z_A(t), t)$ describes the evolution of the orientation of spin A.
The case of particle B is different. B follows a rectilinear trajectory with $y_B(t) = v_y t$, $z_B(t) = z_0^B$ and $x_B(t) = x_0^B$. By contrast, the orientation of its spin moves and it was $\theta^B(t) = \pi - \theta(z_A(t), t)$ and $\varphi^B(t) = \varphi(z_A(t), t) - \pi$.

We can then associate the wave function:

$$\Psi_B(x_B, z_B, t_0 + t + \triangle t) = f(x_B, z_B)\left[\cos\frac{\theta^B(t)}{2}|+_B\rangle + \sin\frac{\theta^B(t)}{2} e^{i\varphi^B(t)}|-_B\rangle\right]. \quad (12)$$

This wave funtion is specific, because it depends upon initial conditions of A (positions and spins). The orientation of spin of the particle B is driven by the particle A *through the singlet wave function.* Thus, the singlet wave function is the actual non-local hidden variable.

Figure 2 presents a plot in the (z, y) plane the trajectories of a set of 5 pairs of entangled atoms whose initial characteristics $(\theta_0^A = \pi - \theta_0^B, z_A^0 = z_B^0)$ have been randomly chosen. The trajectories will therefore depend on both the initial position z^0 and the initial spin orientation θ_0. Since the spin initial orientation are different, trajectories of the A particles may intersect.

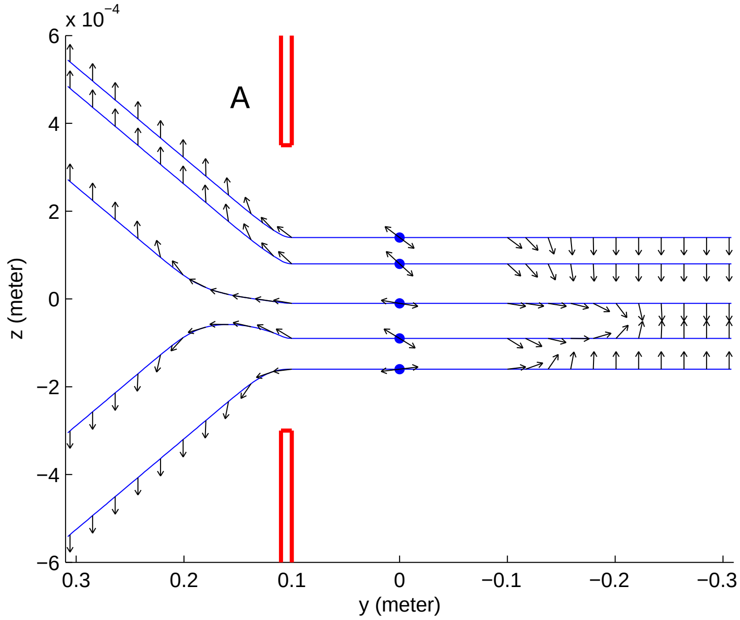

Figure 2: Five pairs of trajectories of entangled particles. Arrows represent the spin orientation.

3.2 Step 2: *"Measurement"* of A spin, and then B spin

Until time $t_0 + t + \triangle t$, we are in the case of step 1. Immediately after the "measurement" of A at the time $t_0 + \Delta t + t$, if the A measurement is $\pm$, the conditional wave function of B is:

$$\Psi_{B/\pm A}(x_B, z_B, t_0 + t + \triangle t) = f(x_B, z_B)| \mp B\rangle. \tag{13}$$

Then particle B is in position (x_0^B, z_0^B).

We are exactly in the case of a particle in a Stern and Gerlach magnet $\mathcal{B}$ which is an angle δ with $\mathcal{A}$.

To measure the spin of B, we refer to the basis $| \pm' B\rangle$. So, after the measurement of B, at time $t_0 + 2(t + \triangle t)$, the conditional wave functions of B are :

$$\begin{aligned}
\Psi_{B/+A}(x'_B, z'_B, t_0 + 2(t + \triangle t)) &= \cos\frac{\delta}{2} f^+(x'_B, z'_B, t)|+'_B\rangle \\
&\quad + \sin\frac{\delta}{2} f^-(x'_B, z'_B, t)|-'_B\rangle, \\
\Psi_{B/-A}(x'_B, z'_B, t_0 + 2(t + \triangle t)) &= -\sin\frac{\delta}{2} f^+(x'_B, z'_B, t)|+'_B\rangle \\
&\quad + \cos\frac{\delta}{2} f^-(x'_B, z'_B, t)|-'_B\rangle
\end{aligned}$$

and we find again the quantum correlations.

4 Conclusion

From the wave function of two entangled particles, we found spins, trajectories and also a wave function for each of the two particles.

In this interpretation, the quantum particle has a local position like a classical particle, but it has also a non local behaviour through the wave function. This is the foundamental innovation introduced by quantum mechanics.

So it is the wave function that create the new, non classical properties. We can keep a view of a local realist world for the particle, but we should add a non-local vision through the wave function.

Therefore the actual hidden variable is the wave function. It is not separable and non-local. As in the Broglie-Bohm interpretation the wave function pilots the particle, it also creates the non separability of two entangled particles.

As we saw in step 1, the non-local influences in the EPR-B experiment only concerns the spin orientation, not the motion of the particles themselves. This is a key point in the search of a physical explanation of non-local influences.

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